QC
916
W3

MINISTRY OF PUBLIC WORKS, EGYPT.

UC-NRLF
$C 189 607

REPORT ON
PSYCHROMETER FORMULÆ
BASED ON OBSERVATIONS IN EGYPT AND THE SUDAN,

BY

E. B. H. WADE, M.A.,

Director of Research.

PHYSICAL DEPARTMENT PAPER No. 2.

CAIRO.
GOVERNMENT PRESS.

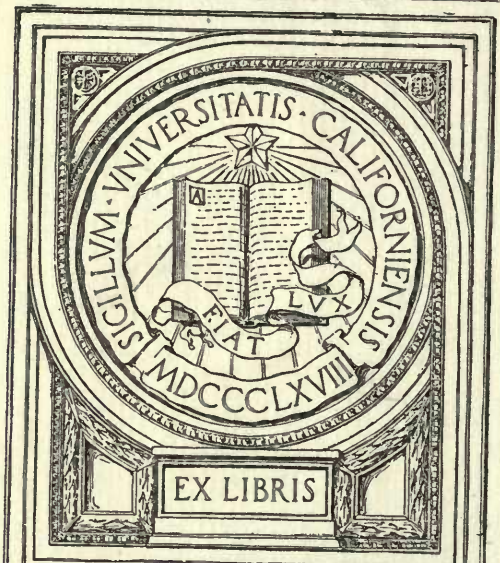
To be obtained, either directly or through any Bookseller,
from the GOVERNMENT PUBLICATIONS OFFICE, Old Ismailia Palace, Sharia Qasr el ʿAini, Cairo.

1920.

PRICE: P.T. 5.

YD 02168

EXCHANGE


SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
MDCCCLXVIII
LVX
FIAT
EX LIBRIS

CAIRO

MINISTRY OF PUBLIC WORKS, EGYPT.

REPORT ON

PSYCHROMETER FORMULÆ

BASED ON OBSERVATIONS IN EGYPT AND THE SUDAN,

BY

E. B. H. WADE, M.A.,

Director of Research.

PHYSICAL DEPARTMENT PAPER No. 2.

CAIRO.
GOVERNMENT PRESS.

To be obtained, either directly or through any Bookseller,
from the GOVERNMENT PUBLICATIONS OFFICE, Old Ismailia Palace, Sharia Qasr el ʿAini, Cairo.

1920.

PRICE: P.T. 5.

QC916
W3

EXCHANGE

CAIRO

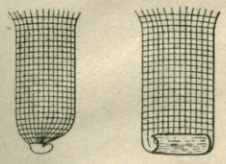


Fig. 1

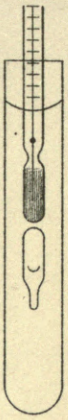
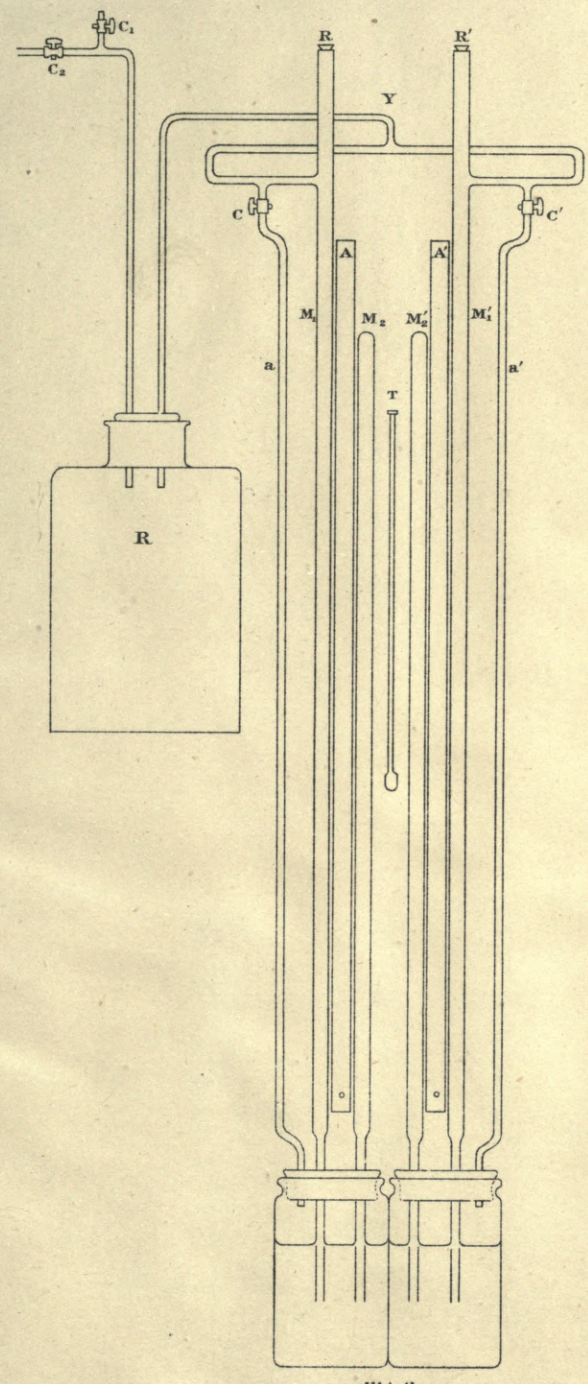


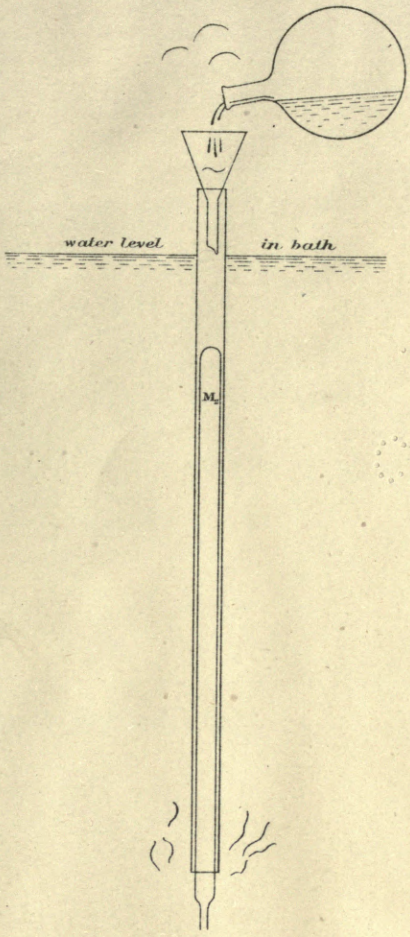
Fig. 2



Printed by the Survey of Egypt, March 1910.

Fig. 3

481495



UNIV. OF CALIFORNIA

Fig. 4

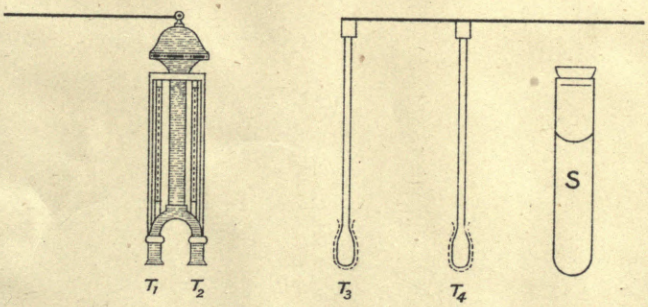


Fig. 5

Printed by the Survey of Egypt, March 1920 (pes)

CONTENTS.

PAGE

I.—Introduction 45

II.—The Sulphuric Acid Method 46

III.—Equipment 46

IV.—Standardization :—

(1) Thermometer Corrections 47

(2) Float Constants 48

(3) Determination of Corrections to Regnault's Tables 50

V.—Method of Observation 64

VI.—Method of Computation... 64

VII.—Some Criticisms 67

VIII.—The Computed Results 67

IX.—Conclusions 72

Govt. Press 7156-1919-300 ex.

UNIV. OF CALIFORNIA

REPORT ON PSYCHROMETER FORMULÆ

BASED ON OBSERVATIONS IN EGYPT AND THE SUDAN.

I.—Introduction.

In Egypt the psychrometer in use for the most careful observations is the well known Assmann aspiration instrument, and the tables used in reducing its readings are Jelinek's (sixth edition). Mr. J. I. Craig, while Director of the Meteorological Service of the Survey Department of Egypt, was led to doubt whether humidities determined as above can be relied upon under all the conditions that are liable to be met with in Egypt and the Sudan, and requested me to examine the question by any methods which appeared suitable. I chose two, *viz.* the dew-point method, and the sulphuric-acid method sketched by me on a former occasion* and developed in the present report.

In order to prevent misunderstanding, some remarks may here be made on the subject of Jelinek's tables. The sixth edition consists of figures calculated under the supervision of Dr. Trabert, with an appendix by Dr. Pernter, and the constants in the formulæ employed are mean values founded on the observations of Regnault, Angot, Blandford, Ferrel, Sworikin, Sprung and Svenson. As the tables are worked out to one place of decimals, and my observations are worked out to the second place, I must be understood when using the expression "correction to Jelinek's tables" to mean the correction applicable to values calculated strictly from the formulæ on which those tables are based. This formula is for convenience referred to in the sequel as Jelinek's formula. The actual entries in the tables sometimes deviate as much as 0·1 millimetre from the computed values owing, presumably, to interpolation error. Again, when speaking of Jelinek's tables, I must not be held to single them out for disparagement. To determine the errors of one set of tables is to determine the errors of all others, and it is precisely because Jelinek's tables are simple, extensive, widely used, and of high accuracy that it happens that the most convenient way of stating the humidity corresponding to a certain psychrometer reading is to say that it is the value given by Jelinek's tables, *plus* the observed corrections to those tables.

In conclusion, I may add that the observations in this report were made while I was travelling about on survey duty, and I had to use such shelters as were available. While it would, no doubt, have been more convenient to have worked in a fixed observatory, I am glad of the opportunity of showing that the methods used are sufficiently simple to be used in the field. The stations occupied were Gîza, Helwân, Kutkata, Marâgha, Tahta, Ashmûn, Minshah in Egypt, and Merowe

* Phil. Mag., April 1902.

in the Sudan. The weather during my brief visit to Merowe was, unfortunately, rather damper than is usual in May, but still it was very dry. The observations and the bulk of the computations had been completed when the war broke out, and it became necessary to hold the work over.

My best thanks are due to my colleagues, Messrs. J. I. Craig, H. E. Hurst, F. Hughes, T. L. Bennett, Dr. J. Ball, and the late Lieut.-Colonel B. F. E. Keeling, who gave me advice or help.

II.—The Sulphuric Acid Method.

The sulphuric-acid method of hygrometry was described in a former paper,* but, as important changes have been introduced, I prefer not to assume that the reader is acquainted with it. Omitting, for the present, practical details, the principle is as follows. Two thermometers are exposed to the air, one having its bulb covered with dry platinum gauze which has been wetted with sulphuric acid solution of known strength. If the tension of aqueous vapour in the air is greater than a_D (the vapour pressure which the given acid has at D the temperature of the air), the acid will absorb water from the air, heat will be developed, and the acid thermometer will rise till an equilibrium temperature is reached. If the tension of aqueous vapour in the air is less than a_D, there will be a fall to a minimum. If the acid bulb gives no thermal reaction on exposure to air, the tension of aqueous vapour in the air must be a_D. But a_D is given by Regnault's observations and my observed corrections. We thus have a standard null method of finding the hygrometric state of the air, or the errors of a given hygrometer or set of tables. In Section V this elementary theory is developed, and leads to an equation in which the principal unknown quantity is, not the vapour tension, but the error of Jelinek's tables. The equation takes account of cases in which the null condition is only approximately realized.

III.—Equipment.

Thermometers.—Those used in the work were made by Fuess, of the type supplied with Assmann's psychrometer. They are graduated to the nearest 0°·2 and estimation to 0°·1 is easy. The bulbs are cylindrical. In all hygrometric observations previous to No. 28/1 (p. 68), the thermometer Fuess No. 1144 was used as T_1 (*see* Sec. V, p. 64). It then received an injury, and was replaced by Fuess No. 2744. Throughout, Fuess No. 1143 was used as T_2, Fuess No. 1136 as T_3, and Fuess No. 351 as T_4.

Platinum Gauze.—This was supplied by Messrs. Johnson Matthey & Co., Ltd., and was used as follows. A piece measuring about 25 × 25 millimetres was cut with a pair of scissors and rolled on to a hard cylinder whose diameter was just less than that of the bulb of T_3. One end of the resulting mantle was neatly folded over as in Figure 1.

The mantle was gently forced on to the bulb of T_3 and, owing to the springiness of the gauze, it became securely fixed without the help of any bandage, but could be removed at will. The bulb of T_4 was provided with a mantle in the same way.

---

* Phil. Mag., April 1902.

Sulphuric Acid.—The acid used was the purest obtainable, being that supplied by Merck for analytical purposes.

Stock Tubes.—From the pure acid seven stock solutions were made. As these would have to be taken with me on my travels, the following conditions had to be complied with: (*a*) the containers must be capable of transport with the minimum risk of breakage; (*b*) their capacity must be small in case a breakage occurred in spite of all precautions; (*c*) if a solution was used up, it must be easy to replace it by one of very nearly identical strength without recourse to troublesome analytical operations; (*d*) to prevent the changes of concentration which are liable to follow the transfer of small quantities of acid from one vessel to another, one and the same container should serve for mixing up solution, checking its concentration, transport, and application direct to the bulb of the thermometer at the moment of observation. Conditions (*a*), (*b*), (*d*) were satisfied by instruments extensively used in Egypt and known in commerce as "albuminimeters." Their rubber stoppers were replaced by glass ones, and they were stored in a very convenient travelling case specially designed to combine safety and portability. Condition (*c*) was met by the use of specific gravity floats, one being permanently located in each of the albuminimeters, or stock tubes as they are termed in the sequel.

IV.—Standardization.

(1) Thermometer Corrections.

In a research of this kind the determination of the thermometer corrections is naturally of great importance. I have, however, pressed the standardization of the thermometers beyond what was strictly necessary, in order that they should constitute a sub-standard for Egypt. For this reason a very elaborate series of comparisons was made between them and the thermometer Baudin No. 15306, which latter has been very fully studied at Breteuil and is the standard of Egypt. The manner in which comparisons of this kind are carried out at Bushey, Breteuil, and similar institutions, has so frequently been described that I need not do so here. I will content myself with saying that the comparison was done under the supervision of the late Lieut.-Colonel Keeling, who, as a former member of the N.P.L. staff, was an expert in such matters. The detailed account of the standardization has been filed in the archives of the Physical Department, Ministry of Public Works, Cairo. I may add that the Fuess thermometers were also furnished with Charlottenburg certificates, and that, as a result of the discussion, we obtain an indirect relation between the gas thermometric standards $\mathfrak{B}$ (Breteuil) and $\mathfrak{C}$ (Charlottenburg), *viz.* $\mathfrak{B} - \mathfrak{C} = -0^{\circ}\cdot01$. In the detailed report I have given reasons for considering that this $-0^{\circ}\cdot01$ is due to a slight difference in the fineness of the ice used on two occasions, it having been discovered, when too late, that such a difference is competent to explain the discrepancy. I have therefore adopted Breteuil as standard and regarded Charlottenburg as confirmatory. The discrepancy is of no importance for the purpose of the present report. Table I gives the corrections arrived at.

TABLE I.—CORRECTIONS $\mathfrak{B}$ — FUESS No. *n*.

	$n =$										
	351	1143	1136	2744	1144		351	1143	1136	2744	1144
10°	—0°·08	—0°·13	—0°·11	—0°·12	—0°·11	23°	—0°·13	—0°·13	—0°·11	—0°·06	—0°·13
11°	—0°·08	—0°·13	—0°·11	—0°·12	—0°·11	24°	—0°·13	—0°·12	—0°·11	—0°·05	—0°·13
12°	—0°·08	—0°·13	—0°·11	—0°·13	—0°·12	25°	—0°·12	—0°·12	—0°·11	—0°·04	—0°·13
13°	—0°·09	—0°·13	—0°·11	—0°·13	—0°·12	26°	—0°·13	—0°·12	—0°·11	—0°·04	—0°·13
14°	—0°·09	—0°·13	—0°·12	—0°·14	—0°·12	27°	—0°·13	—0°·12	—0°·11	—0°·04	—0°·13
15°	—0°·10	—0°·14	—0°·12	—0°·14	—0°·12	28°	—0°·14	—0°·13	—0°·11	—0°·05	—0°·13
16°	—0°·10	—0°·14	—0°·12	—0°·14	—0°·12	29°	—0°·15	—0°·13	—0°·11	—0°·05	—0°·13
17°	—0°·11	—0°·14	—0°·12	—0°·13	—0°·12	30°	—0°·15	—0°·13	—0°·11	—0°·05	—0°·13
18°	—0°·13	—0°·14	—0°·12	—0°·12	—0°·13	31°	—0°·16	—0°·14	—0°·12	—0°·05	—0°·13
19°	—0°·14	—0°·14	—0°·12	—0°·11	—0°·13	32°	—0°·16	—0°·14	—0°·12	—0°·06	—0°·13
20°	—0°·15	—0°·14	—0°·12	—0°·10	—0°·13	33°	—0°·16	—0°·14	—0°·13	—0°·06	—0°·13
21°	—0°·15	—0°·14	—0°·12	—0°·09	—0°·13	34°	—0°·16	—0°·14	—0°·14	—0°·06	—0°·13
22°	—0°·14	—0°·14	—0°·12	—0°·07	—0°·13	35°	—0°·16	—0°·14	—0°·14	—0°·07	—0°·13

(2) FLOAT CONSTANTS.

As explained above, the concentration of solutions was determined by means of floats. The manner in which these were standardized is illustrated by Table II. An acid solution was prepared and introduced into a sample tube containing float F I and a thermometer (*see* Fig. 2).

The contents were cautiously warmed until the float just parted from the thermometer and sank very slowly. The temperature at which this occurred was observed to be 34°·0. The density of this acid was then carefully determined by pyknometer, with the result $d_4^{24\cdot8} = 1{\cdot}31085$, whence (Landolt and Börnstein, 1905, p. 326) the percentage of sulphuric acid is 41·33 per cent. In order to render all our observations comparable, we now find P_{40} the percentage which would have caused the float to sink at 40° instead of 34°·0. The correction is + 0·083 (40 — T), the factor being deduced from the table in Landolt and Börnstein just quoted. The resulting value for P_{40} is 41·83 per cent. All the other values are obtained in precisely the same way.

Confirmation by Analysis.—While my adopted float constants are based on density determinations, there is good confirmation by chemical analysis kindly made by Mr. Hughes. In the earlier titrations by Na_2CO_3 he found percentages slightly lower than mine. He then made gravimetric analyses, weighing as $(NH_4)_2SO_4$, which confirmed my results. He then carefully examined the Na_2CO_3, checking it against $KHSO_4$, a very reliable substance, and found that, though the Na_2CO_3 had been carefully ignited, there was a slight defect of alkalinity in terms of $KHSO_4$. On correcting for this and collecting the results we have the figures in Table III, which support the density determinations in a very satisfactory way.

The finally adopted float constants are shown in Table IV. From these data we can rapidly find the concentration of a solution by the formula

$$P = P_{40} - f(40 - T),$$

T being the observed floating point of the acid whose concentration is sought, and *f* the temperature coefficient of the given float.

TABLE II.—EXAMPLE OF STANDARDIZATION OF FLOATS.

Float.	Temperature at which Flotation occurred = T.	Percentage of H_2SO_4.	Reduction from T to 40° = 0·083 (40—T).	Inferred P_{40}.
F I	34°·0	41·33	+ 0·50	41·83
,	34°·0	41·35	+ 0·50	41·85
,	41°·6	42·00	— 0·13	41·87
,,	43°·0	42·02	—	41·77
		Mean		41·83

TABLE III.—COMPARISON OF CHEMICAL AND PHYSICAL DETERMINATIONS.

Float.	P_{40}.		
	By $(NH_4)_2SO_4$.	By Na_2CO_3.	By Density.
F I	41·65	41·82	41·83
	41·88	41·87	41·85
	—	41·75	41·87
	—	42·02	41·77
F II	—	64·47	64·53
	—	64·48	64·53
	—	64·52	64·50

TABLE IV.—SUMMARY OF ADOPTED FLOAT CONSTANTS.

Float.	P_{40}.	Temperature Coefficient $= f$.
F I	41·83	0·083
F II	64·51	0·076
F III	60·16	0·076
F IV	77·16	0·083
F V	34·45	0·087
F VI	32·40	0·087

(3) DETERMINATION OF CORRECTIONS TO REGNAULT'S TABLES.

Preliminary.—Without going into details of the way in which observations in sulphuric acid hygrometry are reduced (*see* §V), it is evident that the accuracy of the method depends ultimately on that of determinations of the vapour pressure of sulphuric-acid solutions. Such determinations were made by Regnault,* and a large number have been published also by Soret.† A smaller number have been made, for dilute solutions only, by Helmholtz.‡ Comparisons between the above reveal discrepancies; for example, we have

33·10 per cent Acid.

Temperature.	Helmholtz — Regnault.
	Millimetres.
15°	—0·13
20°	—0·16
25°	—0·10

and again

* *See* Landolt & Börnstein's Tables, Ed. 1905, page 166.

† *Ibid.*, page 166.

‡ *Ibid.*, page 167.

64·47 per cent Acid.

Temperature.	Soret — Regnault.
	Millimetres.
10°	0·0
15°	—0·1
20°	—0·1
25°	—0·1
30°	—0·2
35°	—0·2

As the hygrometric method described is competent to detect with certainty changes in the vapour pressure amounting to ± 0·1 millimetre, such uncertainty in the fundamental data could not be tolerated, and it was decided to make new determinations of the vapour pressure of sulphuric acid solutions. It would have been interesting to cover the whole ground studied by Regnault, but the Egyptian Government could not be expected to sanction so extensive a research into a matter which, for the most part, did not interest them, and I was obliged to confine myself to making corrections to Regnault's tables at those concentrations and temperatures which were involved in the humidity observations.

Choice of a Method.—This presented some difficulties. Those methods which make use of the Torricellian space over mercury may be considered as barred, owing to the influence of volatile impurity, though Tamman*, at any rate, seems to have got very good results in this way. Boiling points by such methods as those of Beckmann or Sakurai would require very elaborate apparatus before they could be adapted to work under an atmosphere of four millimetres, and even then success would be doubtful. An attractive method due to Ewan and Ormandy,† which I have used with good results in the past, was ruled out for one reason only. It depends on the observation of the dew point. Now existing humidity tables practically depend on dew points. To check these tables by the sulphuric acid method, and then check the vapour pressure of the sulphuric acid by the dew point method, would be to make the dew point once more the ultimate standard of reference, and nothing would have been gained. Methods depending on the bubbling of air in parallel through water and a solution are tedious and require extreme care. With regard to Helmholtz's method, which depends on the formation of a cloud of mist, theoretical questions are involved, and though no doubt these have received the ablest treatment at the hands of the inventor, there was a possibility that the theory might not be absolutely rigorous for strong solutions. Without asserting that such is the case, I preferred, in making fundamental measurements,

* Mem. Acad. Petersburg (7) 35; 1887.

† J. C. S. 61 (1892) 769.

to employ a perfectly direct method. The vapour-pressure apparatus finally adopted bears a considerable resemblance to that described by Lord Kelvin.* As the resemblance is accidental, it is not surprising that there are also important differences between the two methods. I will describe my procedure, and in doing so will indicate the points where it appears to me to have the advantage over Lord Kelvin's. The diagram (Fig. 3) gives a good idea of the apparatus, but slight changes have been made so that the arrangement should be intelligible from a single drawing. Thus, in the actual apparatus, the T tube Y is not vertical as shown, but lies in a horizontal plane, and the six tubes a, M_1, M_2, M'_2, M'_1, a' are not all in one plane, but tubes a, a' lie somewhat behind the plane of the paper. The whole is immersed in a zinc water bath, measuring about 30 × 30 × 30 inches and stiffened with thick wooden casing, the contents being well stirred by an electrically driven propeller. Tap C_1 communicates with the atmosphere, and C_2 with a vacum pump.

Filling and Deaeration.—The rubber stoppers R and R′ are withdrawn. A thistle funnel with long capillary stem is passed down M_1 to the bottom of B, and boiled-out water is poured in; similarly, boiled-out solution by way of M'_1 into B′. The stoppers are replaced, and the taps C_2, C, and C′ are opened. Air is thus drawn out of M_2, M'_2. On shutting C_2 and opening C_1, liquid takes its place. When as much air as possible has been withdrawn in this way, a glass tube is held as in Figure 4, and boiling water is rapidly poured down it while the pump is kept working. This causes the liquid adhering to the sides of the tube M_2 to boil and sweep out the air. The same treatment is applied to M'_2. All but minute traces of air can be removed in this way, but *some air is always left*; and its effect must be duly corrected for (*see* p. 53).

Before proceeding to make observations, the water bath is raised to a temperature higher than any at which it is proposed to work, and the pressure is reduced as much as possible. This is with a view to boiling out the contents of B and B′, for, however well they were originally boiled out, they will have dissolved some air in the process of exhausting M_2 and M'_2. In spite of these precautions, the method of observing is founded on the assumption that air is never completely removed and that the amount of air in the vacuum is liable to gradual change.

Observation.—The water bath is first adjusted to be steady at the desired temperature. C_1 is opened; a small amount of air is thus admitted, and the liquids retreat to the top of M_2 and M'_2. C_1 is closed and C_2 opened till the liquids have descended again to positions fixed by experience. C_2 is closed, C and C′ are closed, then C_2 re-opened. The menisci in M_2 and M'_2 continue to descend, but only very slowly, and the principal movement is now a rise of liquid in the tubes M_1 and M'_1. Pumping is kept up till this rise reaches points fixed once more by experience. C_2 is now closed, and the final step consists in cautiously opening C to a very slight extent. The menisci in both M_1 and M_2 fall, and can be arrested exactly at the desired point by closing C. The same is done for C′. Having obtained the desired positions, it is very easy, by alternately opening C_2 and C_1 to make the liquids rise and fall in the tubes so as to secure a thorough flushing of the walls. As observing is carried out in accordance with a carefully pre-arranged programme designed to remove systematic error, it is important to have this power to bring the menisci in the tubes

* "Nature," LV, 273.

to a standstill in the neighbourhood of pre-determined readings. Some little experience with the apparatus soon teaches the observer the precise effect of opening a given tap at a given moment, and with this experience it is easy to secure the desired result. A "simple observation" consists in recording the scale readings of the positions of the liquids and thermometer, reading in the following order S_1, S_2, T, S'_2, S'_1, and then in the reverse order. S_1 is the scale reading of the meniscus in M_1, and so on, while T is the thermometer reading. The scale readings are secured with a laboratory telescope, and the thermometer readings with a lens pressed up against the plate-glass window of the water bath.

Influence of Residual Air.—Such a "simple observation" would furnish at once the vapour pressure of a given solution were it not for the presence of traces of air in M_2 and M'_2. In fact if p is the vapour pressure of water, and p' that of acid solution, ρ the density of water, ρ' of solution, we have

$$p - p' = [\rho'(S'_1 - S'_2) - \rho(S_1 - S_2)]\, 0{\cdot}7345 = d \text{ (say)}.$$

To correct for air pressure we require at least three such simple observations. In the first the spaces above water in M_2 and solution in M'_2 are both large, and such an observation is called an AW observation. Next we reduce the space above, say, the water to fifty per cent of its former value, and re-observe. This is called an Aw observation. Finally, we make an aW observation with the space above the acid reduced fifty per cent and that above the water restored to its original volume. Assuming that the mass of air in the tubes M_2 and M'_2 undergoes no change, we have

$$d_0 = d_1 + x'/V'_1 - x/V_1 \quad \text{.....(AW)}$$
$$= d_2 + x'/V'_2 - x/V_2 \quad \text{.....(aW)}$$
$$= d_3 + x'/V'_3 - x/V_3 \quad \text{.....(Aw)}$$

If $V_1 = V_2 = 2V_3$ and $V'_1 = V'_2 = 2V'_3$ exactly,

the three equations reduce to

$$d_0 = 3d_1 - (d_2 + d_3)$$

which is useful as a check.

About fifty preliminary observations were worked out in this way, varying details as much as possible to gain information. The general conclusion was as follows. The quantities x and x' are liable to undergo change. In so far as this change is accidental, we may rely on the mean of a number of observations to give us the correct result. In so far as it is a systematic process, the result will depend on the order in which the observations AW, aW, Aw were made, and the elimination of the time factor is merely a matter of suitably arranging our programme. By taking AW at time t_1, Aw at t_2, aW at t_3, repeating AW at t_4, and using the mean of the two AW observations to give us d_1, we have very nearly the effect of three simultaneous observations at time $t_{2\cdot5}$, but Aw is a little too soon and aW a little too late. If we now repeat the set, but this time take aW at t_2 and Aw at t_3, the mean of the two sets is free from any systematic error due to dependence of x and x' on time. A group, AW, Aw, aW, AW or AW, aW, Aw, AW, constitutes a "complete observation," and we arrange to have about as many of the former as of the latter type in a set. From the time that complete observations were made in the way just described the irregularities of the preliminary series disappeared.

Advantages of the Method selected.—I have already indicated the general considerations which led me to reject various methods already described. The present method has, I think, the convenience of the static methods without their disadvantages. In ordinary static methods a *small* mass of solution is *confined* in the Torricellian space over a *heavy* liquid, *viz.* mercury. The small mass is easily contaminated or modified in concentration; the confinement allows a minute trace of volatile impurity to exercise a prejudicial effect; the heaviness of the manometric liquid is somewhat opposed to sensitiveness. In the present method we have a *large* mass of solution which serves also as a *light* manometric liquid, and, above all, there is *no confinement.* Volatile impurity is free to distil away from the bulk in B and B′. The manipulations which have been described make it easy at any moment to bring the liquid in the tubes M_2 and M'_2 to the same concentration as the bulk in B and B′. Some of these features were anticipated in Lord Kelvin's method. In presuming to criticize the design of so eminent an inventor, I find myself in a position of difficulty from which I only escape by assuming (as I believe I may) that the article just quoted is intended merely as a suggestion from which a practical method might be developed in the light of experience. Let us take the description just as it stands. Lord Kelvin evidently intends to obtain his temperature by local jacketing of the liquid columns (*see* dotted lines in the figure and remarks on operation No. 4). These columns extend downwards into the bulk liquid in the Wolff bottles at temperature of the laboratory, and diffusion currents would disturb the temperature of the menisci, particularly when working below the temperature of the laboratory. The entire apparatus might indeed be placed in a large water bath, but, if so, the whole procedure and design need so much modification that we will not pursue the suggestion, for the difference between one vapour-pressure method and another is more a matter of working details than of fundamental principle. If the operation described as No. 3 were carried out as stated, one of the liquid columns would inevitably boil explosively, and the contents of the two columns would be disastrously mixed. If we stop short of this disaster, air will be incompletely removed. Assuming operations No. 4 and No. 6 to proceed without explosion, we have, in the former, a condensation of solvent into solution, and, in the latter, distillation of solvent away from solution. These phenomena occur locally at a vital point, *viz.* the solution's upper meniscus, whose concentration is thus rendered uncertain. We must therefore supplement Lord Kelvin's operations by operation No. 10, *viz.* "mixing with the stock in the Wolff's bottle." We must open the lower air pump stop-cock till the columns descend, then shut it, and exhaust till they recover their positions. But the liquid in the Wolff's bottles always contains air (dissolved from the air above it, whose presence is indispensable to the working of the method). We are thus obliged to repeat operation No. 6, and so on. Lord Kelvin intends that a long series of observations should be made, preceded by No. 6 and without disturbing the conditions. In that case the concentration at the meniscus and the residual air pressure would undergo gradual change. All these troubles disappear if, instead of attempting to keep the residual air pressure constantly zero, we recognize that both it and the change of concentration are unavoidable errors which must be corrected for. Not only is this course the more accurate, but much time is saved by adopting it. Operation No. 9 gives a means to "facilitate and approximately determine the hydrostatic correction." Such an operation is open to the objections raised against Nos. 4 and 6 and against my suggested No. 10. As the operation is described as "approximate," it would seem better to omit it, but then we are forced to make readings not only to

the upper menisci in the tubes but to the lower ones in the Wolff bottles. Such readings are unsatisfactory. I am not saying that any one of the operations taken alone and for a short space of time would produce much disturbance of air pressure and of meniscus concentration, but if the columns are left undisturbed during a series of observations such changes occur spontaneously. To prevent such change the various operations would have to be kept in constant use, and those which are designed to get rid of air would disturb the meniscus concentration, and the only one I have been able to suggest (No. 10) to get rid of changes of meniscus concentration would re-introduce air. I must emphasize the difference between meniscus concentration and bulk concentration. In any method in which the menisci of water and solution can distil into one another or into a common vacuum, the concentration at the meniscus must be quite uncertain. It will be observed that in my method the menisci in M_2 and M'_2 cannot communicate with one another or with the exhaust. I have not raised any objection which is not the result of my personal observation with analogous apparatus. It may seem unfair to criticize in so much detail what appears to be rather a suggestion than an account of successful observations, but it was necessary, because Lord Kelvin's method is, as far as I know, the only one in which the water and solution are at the same time the manometric liquids—a disposition which has much to recommend it.

Calibration.—The glass scales A and A′, or two precisely similar ones B and B′, are fixed in position on the apparatus by brass clips. The scales were carefully divided for me by Mr. Neumann, using an apparatus improvized for the occasion. The unit selected was approximately 2·11 centimetres, each unit being divided into tenths. These four scales were, each of them, compared by me with a standard Elliott scale divided into millimetres on electrum. Owing to the use of an improvized dividing engine, considerable calibration errors were to be expected. Accordingly, I compared each separate division (0·1 unit) against the standard. The work was gone through twice for each of the four scales, and the mean of the two sets was adopted. As the scales A and A′ were made in one act, their calibration errors should be practically identical, and similarly for B and B′. This check is well satisfied. The final calibration tables show that few divisions are in error by more than 0·1 millimetre, a result very creditable to Mr. Neumann.

Virtual Apex.—In ideal conditions the volumes V and V′ would be given by the differences $Z - S_2$ and $Z' - S'_2$, in which Z and Z′ are the apical readings. Practically, however, the points Z and Z′ must be understood to be the "virtual apices" situated somewhat lower than the true apices. This is due to the curvature of the glass at the apex and to the curvature of the menisci in the tubes. Moreover, if the calibre of the tubes is variable, Z and Z′ must be variable quantities depending on the positions of the menisci. This last refinement, however, fortunately proves to be a superfluous one. In order to determine Z and Z′, the tubes M_2 and M'_2 were removed from the apparatus and placed closed end downwards. Successive charges of mercury were introduced in the way described by Sutton ("Volumetric Analysis," Ed. 10, p. 510). A scale reading of the mercurial meniscus was made each time. If R_1, R_2, R_3 are such readings we can find Z either by

$$Z = R_1 - (R_2 - R_1)$$

or

$$Z = R_1 - 0{\cdot}5\,(R_3 - R_1),$$

the second equation serving to show if Z varies, *i.e.* if the calibre is bad. The following is the result :—

	Water Tube (M_2).			Acid Tube (M'_2).	
	Scale A.	Scale B.		Scale A'.	Scale B'.
$Z = R_1 - (R_2 - R)$ ...	19·22	19·61	$Z' = R_1 - (R_2 - R_1)$	18·88	19·28
$Z = R_1 - \frac{1}{2}(R_3 - R_1)$	19·28	19·68	$Z' = R_1 - \frac{1}{2}(R_3 - R_1)$	18·88	19·28
Mean	19·25	19·64	Mean	18·88	19·28

It will be seen that M_2 shows slight traces of conicality, but, having regard to the fact that Z is required only for correcting for the presence of minute traces of air, a mean value satisfies all requirements. The differences between scale A and scale B are not due to error, but simply due to a difference in the relation between tube and scale in the two cases. In order that the relations between tube and scale might be exactly the same during the determination of Z, and throughout the vapour-pressure work, a diamond scratch was made on each tube and read on each scale. I thus obtained :—

	Water Tube.		Acid Tube.	
	Scale A.	Scale B.	Scale A'.	Scale B'.
Reading of diamond scratch during vapour-pressure observations...	13·70	14·10	13·69	14·10
Reading of diamond scratch during determination of Z and Z'	13·70	14·10	13·69	14·10

These figures show that the necessary constancy was secured.

Concentration of Solutions.—I aimed at making the stock solutions stronger by a few tenths per cent than the mean concentration used in the humidity observations, so as to allow for slight deterioration during the work. A pipette of 20 cubic centimetres capacity had the stem below the bulb drawn into a very long capillary. To the stem above the bulb a long rubber tube and pinch cock were attached. From time to time the observations of vapour pressure were interrupted, the stopper R' was withdrawn, the carefully cleaned and dried pipette was lowered into the liquid in B', and a sample was drawn out. The sample was transferred to one of the sample tubes, and its concentration was determined by the method of flotation. The deterioration of strength, with time, was found to be exceedingly small. A table was prepared giving by interpolation the concentration to be used in the reduction of each vapour-pressure observation. These interpolated concentrations are rounded off to the nearest 0·05 per cent, but the ones actually observed by flotation are given to 0·01 per cent (*see* Table V).

The Actual Observations.—Table VIII gives an example of an actual observation for vapour pressure, *viz.* No. 10/1, made exactly in the way described on page 53.

Reduction of Observations.—The elementary formula for a lowering of vapour pressure is given on page 53. The factor 0·7345 reduces to millimetres of mercury under standard conditions. "Simple observations" belonging to the same set differ amongst themselves by minute amounts in temperature and concentration, and to reduce to a common temperature and concentration we must correct the right-hand side of the equation on page 58 by two small quantities taken from Table VII, which contains other useful data as well. To correct for residual air we have, as explained, to make four simple observations, the first and fourth being combined to give us d_1, the second to give d_2, the third to give d_3. The quantities $Z - S_2$ and $Z' - S'_2$ give us V and V' for each observation, after which we proceed to the reduction of the three simultaneous equations (p. 53).

From d_0 and p, the vapour pressure of pure water, we find p' the vapour pressure of the given solution, and on comparing the latter with Regnault's value we find W — R the observed correction to Regnault. By way of illustration I give in Table VIII the reduction of the complete observation No. 10/1, one of a set of five, made at closely similar concentrations and temperatures.

The concentration and temperature to which all the members of the set are to be reduced are, respectively, 41·90 per cent and 14°80. The values of ρ and ρ' are taken from Table VII. S_1, S_2, S'_1, S'_2 in centimetres are obtained from the observed quantities with the help of the calibration table. T is the observed temperature, *plus* correction in Table I. P is from Table V. The reductions from T to a common temperature, and from P to common concentration, are made with the help of Table V. The values for Z and Z' are converted to centimetres from thosegiven on page 56 by aid of the calibration table. All other complete observations are computed in exactly the same way as No. 10/1, and the results are given in Table IX.

Discussion.—The values of W — R for each acid being collected together the question arises how best they may be used. Take, for instance, the actual results at 41·90 per cent: (*a*) we may regard the differences between the means at 14°·80, 19°·80, etc., as due entirely to accidental error, and adopt the general mean over the whole range; (*b*) we may find the linear equation which best fits the observations; (*c*) we may treat each one as exact, and resort to linear interpolation between them; (*d*) we may find an equation of the second degree to fit the observations. Of these (*a*) has some advantages, but we should have to admit that means of six observations may be in error by as much as 0·13 millimetre, whereas the agreement between the individuals in a group suggests a higher degree of accuracy. Course (*b*) seems reasonable, and leads to the following:—

Temperature.	Observed.	Computed.
	mm.	mm.
14°·80	0·29	0·32
19°·80	0·25	0·22
24°·80	0·07	0·12

It will be seen that " observed " and " computed " differ by no more than we have a right to expect. The remaining courses (*d*) (*e*) would only be justified if we were dealing with a much more extensive series of observations. As it is always difficult to come to an impartial decision in a matter of this sort, I may add that I put the matter before my colleague, Mr. Keeling, who independently came to the same conclusion as I have, *viz.* that it would be advisable to find the linear equation which best fits the observations. Finally, I would say that it makes extremely little difference which course we adopt. The effect of changing from one course to the other would, in the main, be to modify somewhat the residuals in equation 7 (Section VI) without altering the quantity Δ, which is the real object of the research.

Adopted Corrections to Regnault.—Table X shows the finally adopted Regnault corrections. They differ from those in Table IX in that: (*a*) they are smoothed to a linear formula; (*b*) they are stated for round values of temperature, *e.g.* 20°·00 instead of 19°·80, etc.; (*c*) they are brought up to the concentrations required for the reduction of the humidity observations (*see* p. 68). In three cases out of four this last adjustment is either excessively small or zero. In the fourth I have had to estimate what would be the difference between the values which hold good at 34·20 per cent and at 32·00 per cent.* For this purpose I have used the observations at 41·90 per cent and assumed that the Regnault corrections increase regularly with the concentration. It is true the evidence for this assumption is scanty, but, on the other hand, the differences which we have to estimate are of the order 0·05 millimetre, and the hypothesis should be sufficiently accurate for that purpose.

Remarks on the W — R Corrections.—Bearing in mind that Regnault's observations were made in the barometric vacuum, and are thus liable to contamination by volatile impurity, one expects the values of W — R to be throughout negative. Reference to Table IX shows that such is the case. But the values of W — R are much larger than was expected. Comparing my figures with those obtained by the only other authorities available we get :—

Percentage Acid.	Temperature.	W — R.	H — R.	S — R.
32·0	20°	—0·14	—0·16	—
42·0	20°	—0·22	—	—
60·1	20°	—0·81	—	—0·10
64·7	35°	—1·45	—	—0·38

W = Wade ; H = Helmholtz ; S = Soret.

* The reason for this is that the float No. Fv with P_{40} = 32·40 per cent met with an accident before the observations on W — R began. Its place was taken by float No. Fvi whose P_{40} is somewhat higher, *viz.* 34·45 per cent.

Thus Helmholtz's observations support mine, but Soret's support Regnault's. I am not in a position to consult Soret's paper or to say how far his figures are independent of Regnault's.

Assuming my observed vapour pressures at 33·10 per cent to be correct, it is difficult to see why those at 65 per cent should be too low. Volatile impurity would make them too high. Great pains were taken to arrive at the correct concentration. It is proper to observe that, if my concentrations are systematically too high, the W — R values would be explained, but the quantity Δ (*see* Section VI), which is the ultimate object of the investigation, would not be affected, because the concentration is determined in the same way in the Δ observations as in the W — R observations. I mention this merely in the wish to be strictly just to Regnault, and not because I see any prospect of explaining the W — R values as errors in my concentrations.

TABLE V.—(Example). PERCENTAGE OF SULPHURIC ACID AT EACH OBSERVATION OF VAPOUR PRESSURE.

Serial Number of Vapour-Pressure Observation.	Number of Acid Preparation.	Number of Float.	Flotation Temperature.	Percentage of H_2SO_4.	
				By Flotation Observation.	Adopted.
—	4375/071213	Fl.	41°·8	41·98	—
7/1	,,		—	—	42·00
7/2	,,		—	—	42·00
—	,,	,,	41°·6	41·96	—
8/1	,,		—	—	41·95
8/2	,,		—	—	41·95
8/3	,,		—	—	41·95
9/1	,,		—	—	41·95
—	,,	,,	41°·6	41·96	—
10/1	,,		—	—	41·90
10/2	,,		—	—	41·90
10/3	,,		—	—	41·90
11/1	,,		—	—	41·90
11/2	,,		—	—	41·90
—	,,	,,	40°·7	41·89	—
12/1	,,		—	—	41·90
12/2	,,		—	—	41·90
12/3	,,		—	—	41·90
13/1	,,		—	—	41·90
13/2	,,		—	—	41·90
13/3	,,		—	—	41·90
—	,,	,,	40°·8	41·90	—
14/1	,,		—	—	41·90
14/2	,,		—	—	41·90
14/3	,,		—	—	41·90
—	,,	,,	40°·8	41·90	—

TABLE VI.—(Example.) OBSERVATIONS OF VAPOUR PRESSURE OF SULPHURIC ACID SOLUTIONS.
December 13, 1913.

T is reading of thermometer in water bath.
S_1 is scale reading of height of liquid in " open tube " (water side).
S_2 is same for " closed tube " (water side).
S'_1 is same for " open tube " (acid side).
S'_2 is same for " closed tube " (acid side).
Z is scale reading corresponding to " virtual apex."
Z is same for acid side.
S_1 and S_2 are read on scale A ; Z = 19·25.
S'_1 and S'_2 are read on scale A' ; Z' = 18·88.
T is read on thermometer Fuess No. 351.

Number.	Hour.	Acid.	S_1.	S_2.	T	S'_2.	S'_1.
10/1	10-45	4375 / 071213	5·00	7·52	14·92	8·16	3·27
			4·99	7·52	14·92	8·16	3·27
			10·50	13·39	14·85	8·37	3·19
			10·49	13·39	14·85	8·37	3·19
			4·49	7·54	14·90	13·64	8·33
			4·48	7·55	14·93	13·64	8·33
			4·59	7·60	14·88	8·37	3·08
			4·58	7·60	14·90	8·36	3·08

TABLE VII.—(Example). COEFFICIENTS FOR USE IN REDUCING VAPOUR-PRESSURE OBSERVATIONS.

Approximate percentage H_2SO_4	42 per cent.				
Approximate temperature	10°	15°	20°	25°	30°
ρ' = Density of H_2SO_4 solution ...	1·3280	1·3242	1·3204	1·3167	1·3129
ρ = Density of H_2O	0·9998	0·9992	0·9983	0·9971	0·9958
Increment of vapour pressure for for 0°·01 :—					
For H_2O	0·0061	0·0082	0·0108	0·0140	0·0181
For H_2SO_4	0·0034	0·0041	0·0057	0·0075	0·0102
Excess H_2O — H_2SO_4	0·0027	0·0038	0·0051	0·0065	0·0079
Increment vapour pressure per 0·01 per cent H_2SO_4, *minus*	0·0022	0·0029	0·0037	0·0050	0·0063

TABLE VIII.—(Example.) CALCULATION OF p' THE TENSION OF AQUEOUS VAPOUR OVER SULPHURIC ACID SOLUTIONS.

The Calculation is made for Concentration ... ... 41·90 %
,, ,, Temperature ... ... 14°·80

	$\rho' = 1{\cdot}3242$	$\rho = 0{\cdot}9992$		
NUMBER IN REGISTER.	10/1			
	1		2	3
S'_2 stated in centimetres	47·24	47·64	47·66	58·78
S'_1 ,, ,, ...	36·89	36·48	36·71	47·58
ρ' $(S'_2 - S'_1)$	13·70	14·78	14·50	14·83
S_2 cm.	45·86	46·02	58·26	45·90
S_1 ,, ...	40·54	39·66	52·18	39·45
ρ $(S_2 - S_1)$	5·31	6·35	6·07	6·44
T=temperature of observation reduced to gas scale ...	14·82	14·79	14·75	14·82
T—14°·80	+ 0·02	— 0·01	— 0·05	+ 0·02
P=percentage of H_2SO_4	41·90	41·90	41·90	41·90
P—41·90 %	0·00	0·00	0·00	0·00
Reduction to 14°·80	— 0·01	0·00	+ 0·02	— 0·01
,, 41·90 %	0·00	0·00	0·00	0·00
$[\rho' (S'_2 - S'_1) - \rho (S_2 - S_1)]$ 0·7345	6·16	6·19	6·19	6·16
Sum of last three terms = d	6·15	6·19	6·21	6·15
$Z' = 69{\cdot}84$ cm.; $Z' - S_2' = V'$	22·60	22·20	22·18	11·06
$Z = 70{\cdot}61$ cm.; $Z - S = V$	24·75	24·59	12·35	24·71

Equations for Solution.

$$d_0 = d + x'/V' - x/V$$

(1) ... ... $= 6{\cdot}17 + x'/22{\cdot}40 - x/24{\cdot}67$
(2) ... ... $= 6{\cdot}21 + x'/22{\cdot}18 - x/12{\cdot}35$ $\therefore$ $d_0 = 6{\cdot}15$ mm.
(3) ... ... $= 6{\cdot}15 + x'/11{\cdot}06 - x/24{\cdot}71$

		Temperature.	
P = tension of aqueous vapour over pure water at		14°·80 =	12·51
$P' = p - d_0$ = tension of same over H_2SO_4 solution of concentration	41·90 %	14°·80 =	6·36
Same according to Regnault	41·90	14°·80 =	6·66
W — R = observed correction to Regnault's tables for	41·90	14°·80 =	— 0·30

Table IX.

N = serial number of vapour-pressure observation.
T = temperature.
P = percentage of sulphuric acid.
W — R = observed correction in millimetres to Regnault's vapour pressures.

P = 41·90 per cent.

T = 14°·80.		T = 19°·80.		T = 24°·80.	
N.	W — R.	N.	W — R.	N.	W — R.
10/1	— 0·30	7/1	— 0·37	12/1	— 0·21
10/2	— 0·28	7/2	— 0·35	12/2	+ 0·09
10/3	— 0·39	8/1	— 0·17	12/3	— 0·11
11/1	— 0·25	8/2	— 0·26	13/1	— 0·17
11/2	— 0·23	8/3	— 0·26	13/2	— 0·13
—	—	9/1	— 0·22	13/3	+ 0·11
—	—	14/1	— 0·28	—	—
—	—	14/2	— 0·20	—	—
—	—	14/3	— 0·13	—	—
Mean	— 0·29 ± 0·02		— 0·25 ± 0·02		— 0·07 ± 0·04

P = 64·70 per cent.

T = 34°·90.

N.	W — R.	N.	W — R.	N.	W — R.
15/1	— 1·69	16/1	— 1·01	17/1	— 1·48
15/2	— 1·46	16/2	— 1·63	17/2	— 1·42
—	—	—	—	18/2	— 1·49
				Mean ...	— 1·45

TABLE IX (*continued*).

P = 60·50 per cent.

T = 25°·00.		T = 29°·90.		T = 34°·90.	
N.	W — R.	N.	W — R.	N.	W — R.
19/1	—0·91	22/1	—1·01	25/1	—1·21
20/1	—0·92	22/2	—1·10	26/2	—1·36
20/2	—1·01	22/3	—1·28	26/3	—1·00
20/3	—0·88	23/1	—0·91	27/1	—1·13
21/1	—0·83	24/1	—1·10	28/2	—0·93
21/2	—1·03	—	—	—	—
Mean ...	—0·93		—1·08		—1·13

P = 34·20 per cent.

T = 25°·00.		T = 20°·00.		T = 15°·00.		T = 10°·00.	
N.	W — R.	N.	W — R.	N.	W — R.	N.	W — R.
32/1	—0·15	35/1	—0·21	37/1	—0·20	39/1	—0·08
32/3	—0·12	35/2	—0·06	37/2	—0·25	39/2	—0·22
33/1	—0·03	35/3	—0·18	37/3	—0·33	39/3	—0·28
34/1	+0·06	36/1	—0·41	38/1	—0·18	40/1	+0·06
34/2	—0·13	36/2	—0·22	38/2	—0·30	40/2	—0·40
—	—	36/3	—0·19	38/3	—0·16	40/3	—0·23
Mean ...	—0·07		—0·21		—0·24		—0·19

TABLE X.—ADOPTED CORRECTIONS TO REGNAULT'S VAPOUR PRESSURES OF SULPHURIC ACID SOLUTIONS.

Temperature.	Percentage of Sulphuric Acid.			
	32·00	42·00	60·10	64·70
10°	—0·18	—0·42	—	—
15°	—0·16	—0·32	—	—
20°	—0·14	—0·22	—0·81	—
25°	—0·12	—0·12	—0·92	—
30°	—	—0·02	—1·03	—
35°	—	—	—1·14	—1·55

V.—Method of Observation.

Four thermometers are mounted as in the diagram (Fig. 5).

They are referred to in the sequel as T_1 and T_2, T_3, T_4. Of these, T_1 and T_2 are arranged as the dry and wet bulb respectively of an Assmann's psychrometer, while T_3 is freely exposed, except that its bulb is covered with a mantle of platinum gauze, and T_4 is similar to T_3. The stock tube S contains sulphuric acid solution of known strength.

At a given moment the stopper of the tube S is withdrawn; the tube is held below the bulb of T_4; then raised so as to moisten the gauze with acid; then withdrawn and re-stoppered. At the end of the first minute the bulb of T_2 is wetted with water and the clockwork of the aspirator is started to run. At the end of the third, fourth, and fifth minutes all four thermometers are read and means are taken for each thermometer. If it is desired to repeat the observations the platinum gauze of T_4 is removed, washed, ignited, and replaced. The process of observing is then gone through again.

The object of reading the thermometer T_3 is to eliminate the effect of unavoidable radiation. T_1 and T_2 are proof against radiation, but T_4 is not. The following experiment shows, however, that the effect of radiation on T_4 is the same as it is on T_3. T_3 and T_4 were mounted in a good screen, and an observation was carried out exactly as above. Next, powerful solar radiation was turned on by means of a mirror. Both thermometers rose by as much as 0°·8, but the difference between the two was unaffected. We may, then, consider that $T_3 - T_4$ gives us what $T_2 - T_4$ would have been in the absence of radiation.

VI.—Method of Computation.

Elementary Theory.—For simplicity we shall first assume that the observations are made in a screen which is perfectly proof against radiation. Accordingly, T_3 will always give the same reading as T_1, and may for the present be ignored.

Let D, W, A be the readings of T_1, T_2, T_4 respectively. Suppose, first, that the strength of the acid has been so chosen that D — A is exactly zero, and let the corresponding value of W be called W_0. Let p be the pressure of aqueous vapour in the air at the time. Then the error of Jelinek's tables can be obtained as follows:—

On the one hand we have

$$p = a_D \quad \text{.....(1)}$$

where a_D is the vapour pressure of aqueous vapour over the given sulphuric acid solution at the temperature D, and is supposed to be known from Regnault's tables (Landolt and Börnstein, 1905, p. 166) and my corrections (*supra*, p. 64). On the other hand, if Jelinek's tables were correct, we should have

$$p = \text{Arg}\ (W_0 \text{ and } D) \quad \text{.....(2)}$$

If Jelinek's tables are incorrect, the value of p given by (2) will not agree with that given by (1), but we can always find a quantity W'_0 which will reconcile the two values of p, that is to say, W'_0 is such that

$$\text{Arg}\ (W'_0 \text{ and } D) = a_D \quad \text{.....(3)}$$

Putting, now,

$$W'_0 = W_0 + \Delta \quad \text{.....(4)}$$

we can find Δ, the correction which must be applied to an observed wet bulb W_0 before Jelinek's tables will furnish the same tension as equation (1).

Extension.—As the condition that D — A = 0 is a troublesome one to realize in practice, we will extend the theory. Suppose that while D remain contant the air becomes slightly damper : T_1 will rise, and for the same reason T_4 must also rise. We may write

$$W - W_0 = l\,(A - D) \quad \text{.....(5)}$$

where l is a function which we will soon investigate, and therefore by (4)

$$W'_0 - W - l\,(D - A) - \Delta = 0 \quad \text{.....(6)}$$

The restriction that radiation be absent may now be withdrawn, because T_3 and T_4 are equally affected, so that their difference gives the value of D — A as if radiation were absent (*see* Section V).

The Function l.—We cannot hope to arrive at the form of l in a completely *a priori* manner. However, we know that A — D is always of the same sign as $W_0 - W_1$ and that there can be no discontinuity in passing through zero. If, therefore, D — A is kept sufficiently small, it would be permissible to assume l constant. And *a posteriori* we shall find that if we plot our observed values of D — A against W'_0 — W they are as well satisfied by a straight line as we can expect, having regard to experimental error (*see* Residuals, Section VII). Nevertheless, I have preferred not to make such an assumption without a severer test, especially as the condition that D be constant has not been observed. We will therefore treat l, not as a constant, but as a linear function of the other variables, and Δ as a linear function of the temperature whence

$$W'_0 - W - l_M\,[1 + \alpha(D - M) + \beta(D - A)]\,(D - A) - \Delta_M[1 + \gamma\,(D - M)] \quad \text{...(7)}$$

in which M is an arbitrary temperature, preferably the mean value of D.

Trial Solutions.—I have referred the foregoing to Mr. Bennett, who has kindly furnished me with a discussion in which, retaining all terms of second order, he arrives at an expression which, except in its notation, differs only from mine by a term

in $(D - M)^2$. This term amounts to saying that Jelinek's errors vary as $(D - M)^2$. Now while we may not know much *a priori* about *l* we know a good deal *a priori* about Δ. We know that Jelinek's errors are (as nearly as dew-point observations can warrant) zero over the whole known range of humidity. I might well have assumed that Δ was a constant over a limited range, but as an extreme precaution I have introduced the term in D — M. Mr. Bennett, in correspondence, appears inclined to accept this argument, but it was finally agreed that three least square solutions be made, *viz.* :—

Formula I, all first order terms.

Formula II, all first and second order terms, except the one in $(D - M)^2$.

Formula III, all first and second order terms.

The material employed was the group of sixty observations comprised in Tables XII and XIII. His results are :—

	Δ_{20}.	Probable Error of Single Observation.	Probable Error of Δ_M.
I	—0°·03	± 0°·090	± 0·012
II	—0°·03	± 0°·082	± 0°·015
III	—0°·08	± 0°·078	± 0°·017

"Solution I, which has the merit of simplicity, has therefore nothing against it on statistical grounds." With regard to the probable error of single observation, we know *a priori* that it cannot well be less than 0°·1. On the other hand, the more unknowns we introduce the smaller we make it appear. From this point of view, as well as from that of probable error of Δ_M, solution I appears the best, as it certainly is the simplest. Solution III is objected to on the grounds stated above, as well as on the statistical grounds. Besides Mr. Bennett's solutions, I have made one, No. IV, with only one constant (*see* equation 6), and I have cut the group into two halves, one containing all values of D above 20° and the other all below, and solved each by the one constant formula, *viz.* solutions V and VI. The last named have the great advantage of restricting the range of D, and also permit us to find values of Δ at rather different hygrometric conditions from those in Solution IV. Mr. Bennett's solutions also permit this, because they make Δ a function of D — M. The five results are tabulated :—

Solution.	D.	A.	D.	A.	D.	A.
	22·5	22·5	20	20	17·5	17·5
I	+ 0·05		— 0·03		— 0·11	
II	+ 0·05		— 0·03		— 0·11	
IV	—		— 0·03		—	
V	+ 0·09		—		—	
VI	—		—		— 0·05	

It will be noticed that the different formulæ lead to very similar results, at any rate if we round the figures off to the nearest 0°·05. On the other hand, there is a well-marked tendency for Δ to increase with D. I cannot believe this to be wholly real, and think it may be due in part to outstanding error in my temperature coefficient of W — R.

Conclusions.—From the foregoing it seems legitimate to conclude that for the group in question the Jelinek errors are almost negligible, and that a one constant formula is adequate for the reduction of the observations. That course has therefore been adopted in the sequel, and the residuals (*see* Section VII) appear to me fully to justify it.

VII.—Some Criticisms.

In January 1913 I gave a demonstration of the sulphuric acid method to my colleagues in the Survey Department and invited criticism. The criticisms received may well have occurred to my readers also, so I deal with them here.

It was pointed out that, if the acid bulb is moistened with acid of concentration P, and the humidity is such that the acid bulb thermometer rises, the acid must be undergoing dilution. The reply is that the contrary has not been assumed. If D and W are given there is for every value of P (the initial concentration) a corresponding value of D — A, and that is all that is assumed in Section V. Of course, the value of l depends, among many other things, on the rate of dilution, but as we determine l empirically that need not disturb us.

Reference was made to the practice of reading at the third, fourth, and fifth minutes and taking the mean, and I was asked what evidence there is that a steady state is established after the third minute. First, a number of special observations show that after the third minute the acid bulb is extremely steady. Secondly, I have not assumed a steady state. The argument is really just the same as before, *viz.* that the value which D — A attains in four minutes is completely determined by D, W, and P, and that is all that has been assumed in establishing my formula.

It was suggested that the acid bulb might be influenced by ventilation. First, experiments detailed in a former paper prove the contrary,* a result largely due to the fact that D — A can be kept within limits ± 2°. Secondly, the observations in this report were made in very calm air. My object being to determine the errors of Jelinek's tables as applied to Assmann's psychrometer (an instrument independent of ventilation), nothing could have been gained by varying the ventilation.

VIII.—The Computed Results.

The results in Table XI have been obtained by the formula

$$\Delta = W'_0 - W - l\,(D - A).$$

The values in the column "per cent" are obtained from the floating points (*see* Section IV, 2); those under D are the corrected mean readings of T_1 (Section V); under W, of T_2; under D — A, of $T_3 - T_4$; the corrections being those given in Table I. The values of W'_0 are taken from an auxiliary table, which need not be reproduced, as they follow from the definition (Section VI). We may, however, remind the reader that he must take account of barometric pressure. For example, in observation No. 2/2 in Table XI, $W'_0 = 15°{\cdot}69$, because if we enter Jelinek's tables with wet bulb 15·69, dry bulb 21·94, and *barometric pressure*

* "Phil. Mag.," April 1902.

755 millimetres, we get the same tension as we do if we enter Regnault's vapour-pressure tables with concentration 41·95 per cent, temperature 21°·94; and then add my correction, *viz.* — 0·18 (p. 64). The values under $W'_0 - W$ explain themselves. Those under v are "computed *minus* observed." The values of l and Δ are those which best fit the observations.

TABLE XI.

Observations at ... ... { Tahta. Kutkata. Menchawi. Helwân. Gîza.

Mean Conditions D = 14·3; W = 11·4.

$W^0 - W - l(D - A) = \Delta$

Barometric Pressure = 755 mm.

whence $l = 1{\cdot}22$; $\Delta = -0°{\cdot}05$.

Number.	Per cent.	D.	W.	D — A.	W'_0.	$W'_0 - W$.	v.
19/1	32·25	10°·36	9°·07	— 1°·23	7°·77	— 1°·30	— 0°·25
19/2	32·25	9°·39°	8°·27	— 1°·16	6°·86	— 1°·41	— 0°·04
19/3	32·25	9°·82	8°·37	— 1°·03	7°·26	— 1°·11	— 0°·19
19/4	32·25	12°·45	10°·07	— 0°·29	9°·66	— 0°·41	+ 0°·01
19/5	32·25	14°·61	11°·27	+ 0°·41	11°·68	+ 0°·41	+ 0°·04
20/1	32·20	16°·31	12°·57	+ 0°·71	13°·29	+ 0°·72	+ 0°·09
20/2	32·20	17°·64	13°·57	+ 0°·82	14°·49	+ 0°·92	+ 0°·03
20/3	32·20	17°·25	12°·97	+ 1°·25	14°·13	+ 1°·16	+ 0°·31
21/1	32·15	18°·94	12°·00	+ 3 ·18*	15°·69	+ 3°·69	+ 0°·10
21/2	32·15	18°·87	11°·67	+ 3 ·41*	15°·62	+ 3°·95	+ 0°·11
22/1	32·15	13°·61	10°·34	+ 0°·50	10°·75	+ 0°·41	+ 0·°05
22/2	32·15	14°·61	11°·07	+ 0°·64	11°·69	+ 0°·62	+ 0°·11
22/3	32·10	15°·08	11°·27	+ 0°·80	12°·14	+ 0°·87	+ 0°·06
22/4	32·10	15°·71	11°·57	+ 1°·15	12°·72	+ 1°·15	+ 0°·19
24/1	32·10	12°·61	9°·64	+ 0°·21	9°·82	+ 0°·18	+ 0°·03
24/2	32·05	13°·11	9°·80	+ 0°·35	10°·30	+ 0°·50	— 0°·13
24/3	32·05	13°·08	9°·70	+ 0°·35	10°·27	+ 0°·57	— 0°·20
24/4	32·05	13°·31	10°·07	+ 0°·38	10°·49	+ 0°·42	— 0°·01
25/1	32·05	15°·88	11°·47	+ 1°·28	12°·89	+ 1°·42	+ 0°·08
25/2	32·00	15°·95	11°·47	+ 1°·35	12°·96	+ 1°·49	+ 0°·09
25/3	32·00	15°·75	11°·30	+ 1°·25	12°·80	+ 1°·50	— 0°·04
25/4	32·00	15°·78	11°·27	+ 1°·31	12°·83	+ 1°·56	— 0°·02
26/1	31·95	15°·75	11°·77	+ 0°·88	12°·81	+ 1°·04	— 0°·02
26/2	31·95	15°·88	11°·87	+ 0°·98	12°·91	+ 1°·04	+ 0°·10
26/3	31·95	16°·25	12°·07	+ 0°·98	13°·25	+ 1°·18	— 0°·14
26/4	31·95	16°·53	11°·87	+ 1°·38	13°·51	+ 1°·64	— 0°·02
27/1	31·90	13°·35	9°·30	+ 1°·24	10°·57	+ 1°·27	+ 0°·18
27/2	31·90	15°·75	10°·40	+ 2°·04	12°·81	+ 2°·41	+ 0°·02
28/1	32·10	14°·93	9°·90	+ 1°·64	12°·01	+ 2°·11	— 0°·18
28/2	32·05	15°·06	10°·07	+ 1°·57	12°·14	+ 2°·07	— 0°·22
28/3	32·05	15°·53	10°·47	+ 1°·74	12°·57	+ 2°·10	— 0°·05
29/1	32·05	14°·33	9°·30	+ 1°·77	11°·45	+ 2°·15	— 0°·07
29/3	32·05	14°·43	9°·47	+ 1°·64	11°·55	+ 2°·08	— 0°·15
30/1	32·00	11°·08	8°·90	— 0°·36	8°·40	— 0°·50	+ 0°·01
30/2	32·00	10°·78	8°·67	— 0°·50	8°·15	— 0°·52	— 0°·14
30/3	32·00	10°·85	8°·70	— 0°·43	8°·19	— 0°·51	— 0°·06
30/4	31·95	10°·71	8°·47	— 0°·36	8°·04	— 0°·43	— 0°·06
30/5	31·95	10°·45	8°·37	— 0°·43	7°·83	— 0°·54	— 0°·03

* It was not intended that values of D — A exceeding 2° should be observed. These two are due to the fact that the 32·15 per cent acid was selected instead of the 41·95 per cent. The observations are retained, as they illustrate the fact that large values of D — A are unobjectionable.

Table XII.

Observations at ... ... { Kutkata. Marâgha. Akhmîm. Helwân.

Mean Conditions

$W'_0 - W - l(D - A) = \Delta$

whence $l = 1{\cdot}11$; $\Delta = -0{\cdot}05$.

$D = 17{\cdot}5$; $W = 11{\cdot}9$.

Barometic Pressure = 755 mm.

Number.	Per cent.	D.	W.	D — A.	W'_0.	W'_0 — W.	v.
1/2	41·95	18°·20	14°·20	—1°·30	12°·51	—1°·69	+0°·13
1/3	41·95	19°·27	15°·06	—1°·54	13°·42	—1°·64	—0°·12
7/4	41·90	19°·74	14°·67	—0°·94	13°·81	—0°·86	—0°·23
9/3	41·90	17°·48	10°·87	+0°·86	11°·90	+1°·03	—0°·13
9/4	41·90	17°·85	11°·64	+0°·44	12°·22	+0°·58	—0°·14
11/2	41·90	17°·51	12°·70	—0°·79	11°·93	—0°·77	—0°·16
11/3	41·90	18°·90	13°·80	—0°·64	13°·12	—0°·68	—0°·12
12/5	41·95	19°·60	12°·87	+0°·72	13°·70	+0°·83	—0°·08
13/1	41·95	19°·04	12°·67	+0°·45	13°·22	+0°·55	—0°·10
13/2	41·95	18°·44	12°·63	+0°·02	12°·68	+0°·05	—0°·08
13/3	41·95	19°·94	13°·54	+0°·53	13°·99	+0°·45	+0°·09
14/2	41·90	15°·81	12°·04	—1°·48	10°·47	—1°·57	—0°·12
14/3	41·90	16°·55	12°·52	—1°·19	11°·11	—1°·41	+0°·04
14/4	41·90	16°·75	12°·66	—1°·09	11°·28	—1°·38	+0°·13
14/5	41·90	17°·25	12°·99	—1°·09	11°·71	—1°·28	+0°·02
15/1	41·90	18°·24	13°·58	—0°·94	12°·55	—1°·03	—0°·06
15/2	41·90	18°·77	13°·75	—0°·59	13°·01	—0°·74	+0°·03
15/3	41·90	19°·54	14°·21	—0°·56	13°·66	—0°·55	—0°·11
15/5	41·90	14°·91	10°·39	—0°·51	9°·70	—0°·69	+0°·07
16/1	42·00	16°·71	11°·82	—0°·46	11°·23	—0°·59	+0°·03
16/2	42·00	16°·98	11°·96	—0°·29	11°·46	—0°·50	+0°·13
16/3	42·00	17°·08	11°·92	—0°·22	11°·54	—0°·38	+0°·09
16/4	42·00	17°·41	12°·22	—0°·12	11°·83	—0°·39	+0°·21
16/5	41·95	18°·17	12°·26	+0·°19	12°·43	+0°·17	—0°·01
17/1	41·95	18°·44	12°·39	+0°·49	12°·71	+0°·32	+0°·17
17/2	41·95	15°·78	10°·59	—0°·05	10°·44	—0°·15	+0°·05
17/3	41·95	14°·55	9°·99	—0°·40	9°·37	—0°·62	+0°·13
17/4	41·95	12°·91	9°·19	—0°·87	8°·02	—1°·17	+0°·16
18/1	41·95	17°·35	11°·89	—0°·09	11°·78	—0°·11	—0°·04
18/2	41·95	17°·80	12°·12	+0°·12	12°·16	+0°·04	+0°·04
18/3	41·95	18°·10	12°·29	+0°·12	12°·43	+0°·13	—0°·05
18/6	41·90	18°·30	12°·19	+0°·42	12°·60	+0°·41	+0°·01
18/5	41·90	18°·67	12°·09	+0°·86	12°·91	+0°·82	+0°·04
21/3	42·05	19°·20	11°·82	+1°·21	13°·34	+1°·52	—0°·23
23/1	42·05	19°·67	12°·56	+1°·21	13°·73	+1°·17	+0°·12
23/2	42·00	19°·94	12°·62	+1°·34	13°·97	+1°·37	+0°·07
23/3	42·00	19°·77	12°·52	+1°·17	13°·83	+1°·35	0°·00
23/4	41·95	19°·79	12°·66	+1°·20	13°·86	+1°·20	+0°·08

Table XIII.

Observations at ... ... { Tahta. Kutkata. Marâgha.

Mean Conditions

$W'_0 - W - l(D - A) = \Delta$.

$D = 22°{\cdot}2$; $W = 15°{\cdot}9$.

whence $l = 1{\cdot}00$; $\Delta = +0°{\cdot}09$.

Barometric Pressure = 755 mm

Number.	Per cent.	D.	W.	D — A.	W'_0.	$W'_0 - W$.	v.
2/2	41·95	21°·94	16°·86	—1°·09	15°·69	—1°·17	+0°·17
2/4	41·95	25°·04	17°·99	—0°·02	18°·29	+0°·30	—0°·23
3/1	41·95	26°·57	18°·79	+0°·62	19°·60	+0°·81	—0°·10
3/2	41·95	27°·54	18°·76	+1°·69	20°·45	+1°·69	+0°·09
3/3	41·95	23°·47	17°·06	—0°·31	16°·97	—0°·09	—0°·13
4/2	41·95	21°·27	16°·26	—1°·39	15°·12	—1°·13	—0°·17
7/1	41·95	21°·26	15°·09	+0°·29	15°·46	+0°·37	+0°·01
7/2	41·90	20°·97	15°·39	—0°·58	14°·87	—0°·52	+0°·03
7/3	41·90	20°·14	14°·99	—0°·90	14°·15	—0°·84	+0°·03
8/1	41·90	22°·34	14°·67	+1°·43	16°·03	+1°·36	+0°·16
8/2	41·90	22°·10	15°·56	+0°·32	15°·83	+0°·27	+0e·14
8/3	41·90	22°·30	15°·73	+0°·09	16°·00	+0°·27	—0°·11
8/4	41·90	22°·34	15°·53	+0°·45	16°·03	+0°·50	+0°·04
12/1	41·95	22°·10	14°·34	+1°·53	15°·82	+1°·48	+0°·14
12/2	41·95	23°·17	14°·40	+2°·14	16°·72	+2°·32	—0°·09
12/3	41·95	21°·14	13°·44	+1°·46	15°·00	+1°·56	+0°·01
12/4	41·95	20°·34	13°·07	+1°·05	14°·33	+1°·26	—0°·12
13/4	41·90	20°·84	14°·10	+0°·70	14°·76	+0°·66	+0°·13
13/5	41·90	21°·47	14°·43	+1°·03	15°·30	+0°·87	+0°·25
14/1	41·90	22°·04	14°·76	+0°·96	15°·78	+1°·02	+0°·03
15/4	41·90	20°·57	14°·18	+0°·20	14°·54	+0°·36	—0°·07
21/4	42·05	20°·04	12°·36	+1°·40	14°·05	+1°·69	—0°·20

Table XIV.

Observations at Merowe.

Mean Conditions

$W'_0 - W - l(D - A) = \Delta$

$D = 29{\cdot}4$; $W = 14{\cdot}1$.

whence $l = 0{\cdot}87$; $\Delta = -0°{\cdot}08$.

Barometric Pressure = 735 mm.

Number.	Per cent.	D.	W.	D — A.	W'_0.	$W'_0 - W$.	v.
31/1	60·25	31°·94	15°·29	+0°·57	15°·62	+0°·33	+0°·09
31/2	60·25	31°·81	15°·59	+0°·21	15°·51	—0°·08	+0°·18
31/3	60·25	31°·76	15°·59	+0°·17	15°·48	—0°·11	+0°·18
31/4	60·25	32°·64	15°·43	+0°·47	16°·03	+0°·60	—0°·27
31/7	60·25	32°·14	14°·99	+0°·91	15°·71	+0°·72	—0°·01
32/1	60·25	31°·74	15°·19	+0°·37	15°·46	+0°·27	—0°·03
32/2	60·20	31°·22	14°·63	+0°·61	15°·16	+0°·53	—0°·08
32/3	60·20	30°·72	14°·96	—0°·13	14°·84	—0°·12	—0°·07
32/4	60·20	29°·95	14°·96	—0°·55	14°·37	—0°·59	+0°·03

TABLE XIV (*continued.*)

Number.	Per cent.	D.	W.	D — A.	W'_0.	W'_0 — W.	*v.*
32/5	60·20	29°·62	14°·83	— 0°·59	14°·17	— 0°·66	+ 0°·07
32/6	60·20	29°·20	14°·17	— 0°·09	13°·91	— 0°·26	+ 0°·10
32/7	60·20	28°·62	13°·87	— 0°·16	13°·56	— 0°·31	+ 0°·09
33/2	60·20	25°·83	12°·30	— 0°·11	11°·87	— 0°·43	+ 0°·25
33/4	60·15	27°·33	13°·14	— 0°·28	12°·80	— 0°·34	+ 0°·02
33/5	60·15	32°·57	15°·66	+ 0°·77	16°·04	+ 0°·38	+ 0°·21
33/6	60·15	33°·37	16°·06	+ 0°·70	16°·54	+ 0°·48	+ 0°·05
33/7	60·15	34°·24	15°·96	+ 1°·52	17°·09	+ 1°·13	+ 0°·11
34/4	60·00	35°·40	16°·69	+ 1°·62	17°·85	+ 1°·16	+ 0 ·17
34/5	60·00	36°·03	16°·36	+ 2°·09	18°·25	+ 1°·89	— 0°·15
36/1	60·05	27°·26	13°·00	— 0°·25	12°·76	— 0°·24	— 0°·06
36/2	60·05	26°·09	12°·34	— 0°·45	12°·05	— 0°·29	— 0°·18
36/3	60·05	25°·43	12°·14	— 0°·69	11°·64	— 0°·50	— 0°·18
36/4	60·05	22°·47	9°·94	— 0°·35	9°·73	— 0°·21	— 0°·17
36/5	60·05	22°·10	10°·00	— 0°·68	9°·50	— 0°·50	— 0°·17
36/6	60·05	22°·36	9°·84	— 0°·25	9°·66	— 0°·18	— 0°·12
36/7	60·10	23°·85	10°·90	— 0°·28	10°·62	— 0°·28	— 0°·04

TABLE XV.

Observations at Merowe.

Mean Conditions

D = 36°·0 ; W = 16°·2.

$W'_0 - W - l(D - A) = \Delta$.

whence $l = 0{\cdot}95$; $\Delta = -0°{\cdot}08$.

Barometric Pressure = 735 mm.

Number.	Per cent.	D.	W.	D — A.	W_0.	W'_0 — W.	*v.*
34/1	64·85	35°·06	16°·23	— 0°·71	15°·65	— 0°·58	— 0°·17
34/2	64·85	35°·03	16°·36	— 0°·68	15°·63	— 0°·73	+ 0°·00
34/3	64·80	35°·23	16°·59	— 0°·71	15°·76	— 0°·83	+ 0°·08
34/6	64·80	36°·36	16°·86	— 0°·16	16°·43	— 0°·43	+ 0°·20
34/7	64·80	36°·00	16°·46	— 0°·28	16°·22	— 0°·24	— 0°·11
34/8	64·80	36°·63	17°·01	— 0°·18	16°·60	— 0°·41	+ 0°·16
35/1	64·80	36°·06	16°·76	— 0°·35	16°·26	— 0°·50	+ 0°·09
35/2	64·75	35°·83	16°·46	— 0°·55	16°·14	— 0°·32	— 0°·27
35/3	64·75	35°·76	16°·36	+ 0°·02	16°·10	— 0°·26	+ 0°20°
37/1	64·75	35°·03	15°·26	+ 0°·45	15°·66	+ 0°·40	— 0°·06
37/2	64·75	35°·83	15°·53	+ 0°·75	16°·14	+ 0°·61	+ 0°·02
37/3	64·75	35°·76	15°·59	+ 0°·75	16°·10	+ 0°·51	+ 0°·12
37/4	64·70	36°·10	15°·66	+ 0°·59	16°·32	+ 0°·66	— 0°·18
37/5	64·70	36°·73	15°·63	+ 1°·22	16°·70	+ 1°·07	+ 0°·01
37/6	64·70	36°·56	15°·66	+ 1°·02	16°·60	+ 0°·94	— 0°·08

We may summarize the computed results as follows :—

TABLE XVI.

Group.	Dry Bulb.	Wet Bulb.	Correction to Jelinek (applicable to the Wet-bulb Reading).
I ...	14°·3	11°·4	— 0°·05
II ...	17°·5	11°·9	— 0°·05
III ...	22°·2	15°·9	+ 0°·09
IV ...	29°·4	14°·1	— 0°·08
V ...	36°·0	16°·2	— 0°·08

IX.—Conclusions.

1. The summary at the end of Section VIII shows that the observed corrections to Jelinek are very small. But for the third entry in the summary, we might have suspected that there was a small systematic error calling for a correction of about — 0°·07. But there is no ground for distrusting the third entry, and we prefer to conclude that the errors of Jelinek are negligeable over the range studied.

2. This satisfactory conclusion as to Jelinek carries with it the conclusion that Regnault's tables for the vapour pressure of sulphuric acid solutions require very large corrections. This conclusion has caused me much anxiety, but I do not see any alternative. If possible, at some subsequent date, I will re-determine the corrections by a new method, but at present it is out of the question for me to do so.

3. The hygrometric method employed is very delicate, and when $D - A = 0$, I think we could trust the value of Δ to 0°·02. But if D and D — A are free to vary within the limits permitted in this report, the theory, though approximately true, is not rigorous enough to fix the value so closely. The various solutions I to VI of Section V, all about equally plausible, give slightly divergent values of Δ. If I were repeating the research I could easily, with the experience gained, secure groups in which D — A was restricted to + 1°, and the range of D to 5°, and I have no doubt increased accuracy would result. However, the Merowe group No. V, Table XVI, which is by much the most important, leaves very little to be desired in this respect. The range of D and D — A is small, and the residuals are very satisfactory.

UNIVERSITY OF CALIFORNIA LIBRARY,
BERKELEY

THIS BOOK IS DUE ON THE LAST DATE
STAMPED BELOW

Books not returned on time are subject to a fine of 50c per volume after the third day overdue, increasing to $1.00 per volume after the sixth day. Books not in demand may be renewed if application is made before expiration of loan period.

MAY 11 1920

50m-7,'27

481495

QC916
W3

UNIVERSITY OF CALIFORNIA LIBRARY